Adv. appl. Clifford alg. *Online First*

DOI 10.1007/s00006-003-0000

Advances in Applied Clifford Algebras

On Hyperbolic Function Theory

Sirkka-Liisa Eriksson and Heinz Leutwiler

Abstract. The hyperbolic version of the standard Clifford analysis will be considered. In this modification the power function x^m becomes a solution. In more details, the Dirac operator $Df = \sum_{i=0}^{n} e_i \frac{\partial f}{\partial x_i}$ with $e_0 = 1$, defined with respect to the Clifford algebra $C\ell_n$, is replaced by the operator $M_k f(x) = Df(x) + \frac{k}{x_n} Q' f(x)$, where $'$ denotes the main involution in $C\ell_n$ and Qf is given by the unique decomposition $f(x) = Pf(x) + Qf(x)e_n$ with $Pf(x), Qf(x) \in C\ell_{n-1}$. The operator M_k $(k \in \mathbb{R})$ will mainly be considered for $k = 0$, $k = n-1$ and $k = 1-n$. In case $k = 0$ the equation $M_k f = 0$ yields the well-known monogenic functions, in case $k = n-1$ one obtains the so-called hypermonogenic functions introduced in [5]. Besides M_k we also study the operator $\overline{M_k} M_k = M_k \overline{M_k}$, a natural generalization of the Laplace operator $\triangle$. Solutions of the equation $\overline{M_{n-1}} M_{n-1} f = 0$ are called hyperbolic harmonic functions. The main goal of this article is to give integral representaions for hypermonogenic and hyperbolic harmonic functions in the upper half space $\mathbb{R}^{n+1}_+$.

Mathematics Subject Classification (2000). Primary 30G35; Secondary 30A05.

Keywords. Monogenic, hypermonogenic, Dirac operator, hyperbolic metric.

Sirkka-Liisa Eriksson
Institute of Mathematics
Tampere University of Technology
P.O. Box 553
FI - 33101 Tampere, Finland
e-mail: `sirkka-liisa.eriksson@tut.fi`

Heinz Leutwiler
University of Erlangen-Nürnberg
Department of Mathematics
Bismarckstrasse $1\frac{1}{2}$
D-91054 Erlangen, Germany
e-mail: `leutwil@mi.uni-erlangen.de`

Received: October, 2005
Accepted: June, 2006