

# Parameter Estimation from Uncertain Data in Geometric Algebra

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**Abstract.** We show how standard parameter estimation methods can be applied to Geometric Algebra in order to fit geometric entities and operators to uncertain data. These methods are applied to three selected problems. One of which is the perspective pose estimation problem. We show experiments with synthetic data and compare the results of our algorithm with standard approaches.

In general, our aim is to find multivectors that satisfy a particular constraint, which depends on a set of uncertain measurements. The specific problem and the type of multivector, representing a geometric entity or a geometric operator, determine the constraint. We consider the case of point measurements in Euclidian 3D-space, where the respective uncertainties are given by covariance matrices. We want to find a best fitting circle or line together with their uncertainty. This problem can be expressed in a linear manner, when it is embedded in the corresponding conformal space. In this space, it is also possible to evaluate screw motions and their uncertainty, in very much the same way.

The parameter estimation method we use is a least-squares adjustment method, which is based on the so-called *Gauss-Helmert model*, also known as *mixed model with constraints*. For this linear model, we benefit from the implicit linearization when expressing our constraints in conformal space. The multivector representation of the entities we are interested in also allows their uncertainty to be expressed by covariance matrices. As a by-product, this method provides such covariance matrices.

**Keywords.** Geometric Algebra, conformal space, parameter estimation, least squares adjustment, pose estimation, fitting, Mahalanobis distance.

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