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Clifford Fourier Transform on Multivector Fields and Uncertainty Principles for Dimensions $n = 2 \pmod 4$ and $n = 3 \pmod 4$

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Abstract. First, the basic concepts of the multivector functions, vector differential and vector derivative in geometric algebra are introduced. Second, we define a generalized real Fourier transform on Clifford multivector-valued functions ($f : \mathbb{R}^n \to Cl_{n,0},\ n = 2, 3 \pmod 4$). Third, we show a set of important properties of the Clifford Fourier transform on $Cl_{n,0},\ n = 2, 3 \pmod 4$ such as differentiation properties, and the Plancherel theorem, independent of special commutation properties. Fourth, we develop and utilize commutation properties for giving explicit formulas for $f\boldsymbol{x}^m$, $f\,\nabla^m$ and for the Clifford convolution. Finally, we apply Clifford Fourier transform properties for proving an uncertainty principle for $Cl_{n,0},\ n = 2, 3 \pmod 4$ multivector functions.

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