

Isometric Action of $SL_2(\mathbb{R})$ on Homogeneous Spaces

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Abstract. We investigate the $SL_2(\mathbb{R})$ invariant geodesic curves with the associated invariant distance function in parabolic geometry. Parabolic geometry naturally occurs in the study of $SL_2(\mathbb{R})$ and is placed in between the elliptic and the hyperbolic (also known as the Lobachevsky half-plane and 2-dimensional Minkowski half-plane space-time) geometries. Initially we attempt to use standard methods of finding geodesics but they lead to degeneracy in this setup. Instead, by studying closely the two related elliptic and hyperbolic geometries we discover a unified approach to a more exotic and less obvious parabolic case. With aid of common invariants we describe the possible distance functions that turn out to have some unexpected, interesting properties.

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