

Quaternions and Small Lorentz Groups in Noncommutative Electrodynamics

V. M. Red'kov and E. A. Tolkachev

Abstract. Non-linear electrodynamics arising in the frames of field theories in noncommutative space-time is examined on the base of quaternion formalism. The problem of form-invariance of the corresponding nonlinear constitutive relations governed by six noncommutativity parameters θ_{kl} or quaternion $\underline{K} = \underline{\theta} - i\epsilon$ is explored in detail. Two Abelian 2-parametric small groups, $SO(2) \otimes SO(1.1)$ or T_2 , depending on invariant length $\underline{K}^2 \neq 0$ or $\underline{K}^2 = 0$ respectively, have been found. The way to interpret both small groups in physical terms consists in factorizing corresponding Lorentz transformations into Euclidean rotations and Lorentzian boosts.

In the context of general study of various dual symmetries in noncommutative field theory, it is demonstrated explicitly that the non-linear constitutive equations under consideration are not invariant under continuous dual rotations, instead only invariance under discrete dual transformation exists.

Keywords. Noncommutative space-time, quaternion formalism, Maxwell electrodynamics, constitutive relations, residual Lorentz symmetry, dual symmetry.

V. M. Red'kov and E. A. Tolkachev
B.I. Stepanov Institute of Physics
National Academy of Sciences of Belarus
68 Nezavisimosti ave.,
Minsk, 220072 BELARUS
e-mail: v.redkov@dragon.bas-net.by

Received: July 30, 2008

Accepted: September, 2008