Adv. appl. Clifford alg. *Online First*

DOI 10.1007/s00006-003-0000

Advances in Applied Clifford Algebras

Solution of Initial Value Problems of Cauchy-Kovalevsky Type in the Space of Generalized Monogenic Functions

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Abstract. This paper deals with the initial value problem of the type

$$\partial_t u(t,x) = \mathcal{L}u(t,x), \quad u(0,x) = u_0(x) \tag{0.1}$$

where $t \in \mathbb{R}_0^+$ is the time, $x \in \mathbb{R}^{n+1}$, $u_0(x)$ is a generalized monogenic function and the operator $\mathcal{L}$, acting on a Clifford-algebra-valued function

$$u(t,x) = \sum_B u_B(t,x)e_B$$

with real-valued components $u_B(t,x)$, is defined by

$$\mathcal{L}u(t,x) := \sum_{A,B,i} c_{B,i}^{(A)}(t,x)\partial_{x_i} u_B(t,x)e_A + \sum_{A,B} d_B^{(A)}(t,x)u_B(t,x)e_A + \sum_A g_A(t,x)e_A.$$

We formulate sufficient conditions on the coefficients of the operator $\mathcal{L}$ under which $\mathcal{L}$ transforms generalized monogenic functions again into generalized monogenic functions. For such an operator the initial value problem (0.1) is solvable for an arbitrary generalized monogenic initial function u_0 and the solution is also generalized monogenic for each t.

Mathematics Subject Classification (2000). Primary 35F10; Secondary 30G35.

Keywords. Initial value problems, Cauchy-Kovalevsky theorem, interior estimates, generalized monogenic functions, associated differential operators.

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Received: October 6, 2008
Revised: June 12, 2009
Accepted: August 13, 2009